MULTIFRACTAL ANALYSIS OF HOURLY WIND SPEED RECORDS
IN PETROLINA, NORTHEAST BRAZIL

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ABSTRACT: We investigate multifractal properties of hourly wind speed records from Petrolina, PE during the period 2008-2011. We use Multifractal detrended fluctuation analysis (MF-DFA), method that was successfully applied to quantify multifractality in non stationary temporal series. Our results show that temporal series of average wind speed and maximum wind speed exhibit multifractal properties. We find that both small and large fluctuations display persistent long term correlations indicated by the values of generalized Hurst exponents, which are above 0.5 the value that characterizes uncorrelated regime. The maximum wind speed shows stronger multifractality then average wind speed indicated by larger width of multifractal spectrum. After shuffling the series, we find that for maximum wind speed multifractality is due to a broad probability density function, while for average wind speed the multifractality arises from both a broad probability density function and long-term temporal correlations.

KEY-WORDS: Wind speed; multifractals; MF-DFA.

1 Introduction

Wind energy plays a strategic role in Brazil’s efforts for sustainable development. Brazil is one of thirteen countries involved in the Solar and Wind Resource Assessment (SWERA) project, designed to provide a reliable database in solar and wind energy resources, together with socio-economic, infrastructure and environmental information, that enable policy makers to evaluate potential for investments in new renewable energy technologies (Martins et al., 2007). The implementation of such technologies will facilitate energy supply in remote areas as in the Amazon region and islands, regulate energy production during dry season and reduce greenhouse gas emissions to the atmosphere, by reducing the fossil fuel consumption. Brazil’s onshore wind-power technical potential is estimated to be about 143 GW (at 50m height) out of which 52% of total potential is located in Northeast Brazil and 21% in the Southeast. Brazil’s wind energy production has increased from 22 MW in 2003 to 602 MW in 2009 as the result of implementation of the government Program for Incentive of Alternative Electric Energy Sources (Programa de Incentivo às Fontes Alternativas de Energia Elétrica - Proinfa),
which was created in 2002 to stimulate the electricity generation from wind power, biomass, and small hydroelectric plants (Dutra and Szklo, 2008). There is a projection for further expansion from approximately 1 GW today to 7 GW in 2014 (Oebels and Pacca, 2013). Brazilian wind potential can contribute significantly to the electricity supply, especially in the Northeast Brazil, where temporal variation of wind energy potential shows complementarity with the flows of the San Francisco River (de Faria et al., 2011). The city of Petrolina is located in semi-arid region of Northeast Brazil at border between Pernambuco and Bahia states. The localization is considered as one of the most promising for the wind energy production in the region (da Silva et al., 2002). The installation of wind farm at nearby Casa Nova with 180MW is in process while three wind farms at Sênto Se with 98.7 MW are already in operation (Chesf, Relatorio de Administraçao 2012, www.chesf.gov.br/).

The evaluation of wind power potential requires careful statistical analysis of mean wind speed and its frequency distribution (de Araujo Lima and Bezerra Filho, 2010; Pimenta et al., 2008; Shipkovs et al. 2013). However, the biggest challenge in integrating wind power into the electric grid is its intermittency and persistence due to temporal and spatial variability of wind in large range of scales (Leahy and McKeogh, 2013; Bakker and Hurk, 2012). Since Haslett and Raftery showed that spatio-temporal variability of wind speed can be modeled using long-memory processes (Haslett and Raftery, 1989), it was recognized that besides the temporal distribution of wind speed and the probability distribution of duration of wind speed episodes, the investigation of internal structure of wind speed dynamics can provide valuable information about underlying stochastic processes that generate temporal variability of wind speed which is crucial for development and evaluation of reliable theoretical and computational predictive models. As a natural process of turbulence wind is the most complex weather variable with specific properties as long-range spatial and temporal correlations and fractal and multifractal fluctuations dynamics (Govindan and Kantz, 2004; Kocak, 2009; Telesca and Lovallo, 2011; de Oliveira Santos et al., 2012; Calif and Schmitt, 2014). Recent studies (Kavassari and Nagarajan, 2005; Feng et al., 2009) show that the complex structure of wind speed temporal series may be modeled as multiplicative cascade processes, similarly to other atmospheric and geophysical phenomena such as rainfall, stream flow and solar wind (Kantelhardt et al., 2006; Szczepaniak and Macek, 2008). In order to provide the information about the nature of underlying stochastic process that generate wind speed temporal variability at the location of Petrolina, which is crucial for development and evaluation of new more reliable prediction models, in this work we study multifractal properties of hourly wind speed temporal series from this location. We apply Multifractal detrended fluctuation analysis (MF-DFA) method (Kantelhardt et al., 2002) which was designed to quantify multifractality in non stationary temporal series.

In the next section we present data and methodology, then we present the results with the accompanying discussion, and finally we draw the conclusions.

2 Data and methodology

The data used in this work are provided by Brazilian National Institute of Meteorology (Instituto Nacional de Meteorologia – INMET) and can be found at the site http://www.inmet.gov.br/. We chose hourly wind speed time series, recorded by automatic
meteorological station in Petrolina (latitude 9.38 degrees South, longitude 40.00 degrees West, altitude 370.46m) during the period 2008-2011. The raw time series (about 24000 hourly observation data points) for the average and maximum wind speed are presented on Figure 1.

![Figure 1](image_url) - The average and the maximum hourly wind speed data for the Petrolina station.

2.1 Multifractal detrended fluctuation analysis (MF-DFA)

Multifractal time series are characterized by a hierarchy of scaling exponents corresponding to different scaling behavior of many subsets of a series (Kantelhardt et al., 2002). Several methods have been proposed for non-stationary processes, in this work we use Multifractal detrended fluctuation analysis (MF-DFA) (Kantelhardt et al., 2002) that has been successfully applied in various phenomena such as physiological signals (Dutta et al., 2010), hydrological processes (Kantelhardt et al., 2006), geophysical data (Telesca and Lapenna, 2006), forest fires records (de Benicio et al., 2013) and financial time series (Oh et al., 2012).

The MF-DFA method proceeds as follows (Kantelhardt et al., 2002): (i) Integrate the original temporal series \( x(i) \), \( i = 1, \ldots, N \) to produce \( y(k) = \sum_{i=1}^{k} [x(i) - \langle x \rangle] \) where \( \langle x \rangle \) is the mean value of \( x(i) \) and \( k = 1, \ldots, N \). (ii) Divide the integrated series \( y(k) \) into \( N_n = \text{int}(N/n) \) non-overlapping segments of length \( n \) and calculate the local trend \( y_s(k) \) (from a \( m \)th order polynomial regression) in each segment \( s = 1, \ldots, N_n \). (iii) Calculate the detrended variance of each segment (by subtracting the local trend) and average over all segments to obtain the \( q \)th order fluctuation function:

\[
F_q(n) = \left( \frac{1}{N_n} \sum_{s=1}^{N_n} \left[ \sum_{k=(s-1)n+1}^{sn} [y(k) - y_s(k)]^2 \right] \right)^{1/q},
\]

where \( q \) can take any real value except zero. For \( q = 0 \), \( F_q(n) \) is calculated as \( [F_q(n) + F_{-q}(n)]/2 \), where \( \varepsilon \rightarrow 0 \) (e.g. \( \varepsilon = 0.001 \)). (iv) Repeat this calculation for many different box sizes \( n \). If long-term correlations are present in original series \( F_q(n) \) should increase with \( n \) as a power law \( F_q(n) \sim n^{h(q)} \), where the generalized Hurst exponent \( h(q) \) is calculated as the slope of the linear regression of \( \log[F_q(n)] \) versus \( \log(n) \). For
monofractal processes $h(q) = \text{const}$, for multifractal time series $h(q)$ is a decreasing function of $q$ and describes the scaling of large (small) fluctuations for positive (negative) values of $q$. The exponent $h(q)$ relates to the classical multifractal exponent defined by the standard partition multifractal formalism as $\tau(q) = q h(q) - 1$. The exponent $\tau(q)$ (also called Rényi exponent) is a linear function for monofractal signals and a nonlinear one for multifractal signal (Kantelhardt et al., 2002). Multifractal series are also described by the singularity spectrum $f(\alpha)$ through the Legendre transform

$$a(q) = \frac{d\tau(q)}{dq},$$

$$f(\alpha) = q\alpha - \tau(q),$$

where $f(\alpha)$ denotes the fractal dimension of the series’ subset characterized by the Hölder exponent $\alpha$. For monofractal signals, the singularity spectrum produces a single point in the $f(\alpha)$ plane, whereas multifractal processes yield a humped function (Kantelhardt et al., 2002).

Multifractality in a time series may be caused by: (i) a broad probability density function for the values of the time series; and (ii) different long-term correlations for small and large fluctuations. The type of multifractality can be determined by analyzing the corresponding randomly shuffled series. The shuffled series from multifractals of type (ii) exhibit simple random behavior with $f(\alpha)$ reduced to a single point, while for multifractals of type (i) the multifractal spectrum is not changed. If the shuffled series demonstrates weaker multifractality than the original one, both kinds of multifractality are present (Kantelhardt et al., 2002).

The properties of the multifractal spectrum can be used to measure the complexity of the time series. We fit the singularity spectra to a fourth degree polynomial

$$f(\alpha) = A + B(\alpha - \alpha_0) + C(\alpha - \alpha_0)^2 + D(\alpha - \alpha_0)^3 + E(\alpha - \alpha_0)^4,$$

and calculate the multifractal spectrum parameters: position of maximum $\alpha_0$; width of the spectrum $W = \alpha_{\text{max}} - \alpha_{\text{min}}$, obtained from extrapolating the fitted curve to zero; and skew parameter $r = (\alpha_{\text{max}} - \alpha_0)/(\alpha_0 - \alpha_{\text{min}})$ where $r = 1$ for symmetric shapes, $r > 1$ for right-skewed shapes, and $r < 1$ for left-skewed shapes.

Roughly speaking, a small value of $\alpha_0$ suggests the underlying process is more regular in appearance. The width of the spectrum $W$ measures the degree of multifractality in the series: larger width means stronger multifractality and “richer” structure of the series.

The skew parameter $r$ determines which fractal exponents are dominant: fractal exponents that describe the scaling of small fluctuations for right-skewed spectrum or fractal exponents that describe the scaling of large fluctuations for left-skewed spectrum. These parameters can be used to measure the complexity of the series: a signal with a high value of $\alpha_0$, a wide range $W$ of fractal exponents, and a right-skewed shape $r > 1$ may be considered more complex than one with opposite characteristics (Shimizu et al., 2002).
3 Results and discussion

In order to make sure that the seasonality does not affect the multifractal analysis, we first normalize the original series $x(t)$ by calculating anomalies

$$X(t) = \frac{x(t) - \langle x(t) \rangle}{\sigma}$$

where $\langle x(t) \rangle$ is the mean hourly wind speed calculated for each hour, and $\sigma$ is the corresponding standard deviation (also calculated for each hour). The results of MF-DFA analysis for average and maximum wind speed are presented on Figures 2, 3, and 4.

**Figure 2** - Fluctuation function $F_q(n)$ versus box size $n$ on double logarithmic scale, for different values of $q$ from $q = -10$ to $q = 10$ with a step of $\Delta q = 1$ (from bottom, to top).

**Figure 3** – Generalized Hurst exponent $h(q)$ and Rényi exponent $\tau(q)$ for the average and maximum velocity time series, for different values of $q$.

It is seen from Figure 2 that within the scaling region of approximately 10 days to 6 months fluctuation function $F_q(n)$ displays linear behavior on log-log scale for $q = -10, ..., 10$, indicating multifractal behavior of both hourly average wind speed and maximum wind speed temporal series. The generalized Hurst exponent $h(q)$ is a decreasing function and Rényi exponent $\tau(q)$ shows non-linear behavior as seen from
Figure 3 which is typical behavior for multifractal processes. The non trivial multifractal spectrum presented on Figure 4, confirms that both average hourly wind speed and maximum hourly wind speed at the location of Petrolina belong to the class of multifractal processes.

Figure 4 – The $f(\alpha)$ spectrum together with regression curves to the fourth order polynomial form.

We observe from Figure 4 that the values of Hölder exponent $\alpha$ are greater than 0.5 indicating that both small and large fluctuations exhibit persistent properties for average wind speed as well as for maximum wind speed. The width of $f(\alpha)$ spectrum is larger for maximum speed indicating a higher degree of multifractality. The measures of complexity $(\alpha_o,W,r)$ shown in Table 1 reveal specific properties of analyzed processes: (i) The position of maximum of $f(\alpha)$ spectrum (that approximates the overall Hurst exponent) for average speed and for maximum speed approaches a pink-noise regime $\alpha_o \to 1$ indicating persistent temporal fluctuations. (ii) The width $W$ of the multifractal spectra is greater for maximum speed indicating stronger multifractality and higher complexity. (iii) The values of asymmetry parameter $r$ reveal that for maximum wind speed the multifractality is more influenced by the scaling of large fluctuations (left skewed spectrum) while for average wind speed small fluctuations contribute more to multifractality (right skewed spectrum).

<table>
<thead>
<tr>
<th>Average</th>
<th>Maximum</th>
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<tbody>
<tr>
<td>$\alpha_o$</td>
<td>$W$</td>
</tr>
<tr>
<td>0.92</td>
<td>0.24</td>
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In order to verify whether the multifractality of the wind speed time series stems from a broad probability distribution, from the correlations within the time series, or both, we next shuffle the time series of average and maximum wind speed and then apply the MF-DFA analysis again: the shuffling procedure performed $10000 \times N$ transpositions on
each series and was repeated 100 times with different random number generator seeds, in order to find the mean shuffled series spectrum, together with standard deviation.

The multifractal spectra of original and shuffled series are shown in Figure 5. The width of multifractal spectrum of original and shuffled series is shown on Table 2. We find that for the maximum wind speed multifractality is due to a broad probability density function (the width of the spectrum is unaffected by shuffling), while for the average wind speed the multifractality arises from both a broad probability density function and long-range temporal correlations (the width of the spectrum decreases after shuffling).

![Figure 5 – Multifractal spectrum $f(\alpha)$ for original and shuffled data.](image)

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Maximum</th>
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<tr>
<td></td>
<td>$W$</td>
<td>$W_s$</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>0.51</td>
</tr>
</tbody>
</table>

**Table 2** The width of the $f(\alpha)$ spectrum for original data ($W$) and the shuffled data ($W_s$).

**Conclusions**

In this work we investigate multifractal properties of hourly wind speed records from Petrolina, PE during the period 2008-2011 by applying Multifractal detrended fluctuation (MF-DFA) method. We find that both processes follow multifractal dynamics characterized by different scaling behavior of small and large fluctuations and require a hierarchy of scaling exponents resulting in non trivial multifractal spectrum. We find that (i) both small and large fluctuations display persistent long term correlations for average wind speed as well for maximum wind speed (ii) The maximum wind speed shows stronger multifractality then average wind speed indicated by larger width of multifractal spectrum. (iii) After shuffling the series, we find that for maximum speed the width of the spectrum stays unaffected indicating that multifractality is due to a broad probability density function, while for average wind speed the width of the spectrum decreases indicating that the multifractality arises from both a broad probability density function and long-range temporal correlations.
The current manuscript deals with long-term correlations of wind records, and as such it is not meant to directly affect the production of wind energy; rather, it is meant to contribute to a better understanding of the phenomenon in the particular area under study. Practical use by the renewable energy community should benefit from such studies in the long term, in the sense that any future "microscopic" statistical models should reflect long term correlations observed in real data, thus leading to more reliable predictions.

Finally, recent studies (Kavassari and Nagarajan, 2005; Feng et al., 2009) indicate that the complex structure of wind speed temporal series may be modeled through multiplicative cascades. Our current results may serve as a starting point for constructing and testing of various such (multifractal cascade) models, that may in turn capable of reproducing important features of wind temporal variability, such as persistence, intermittency and extreme values.

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RESUMO: Neste artigo aplicamos o método Multifractal detrended fluctuation analysis (MF-DFA) para analisar a dinâmica das séries temporais horárias da velocidade e da rajada do vento em Petrolina, Brasil, durante o período 2008-2011. Os resultados mostraram que series temporais da velocidade e da rajada possuem propriedades multifractais, com correlações de longo alcance persistentes para ambas pequenas e grandes flutuações. A serie da rajada mostra a multifractalidade mais forte indicada pela maior largura do espectro multifractal. Analisando as series randomizadas, encontramos que a função densidade de probabilidade dos valores contribui para a multifractalidade da rajada, enquanto no caso da velocidade a multifractalidade é causada pela função densidade de probabilidade e pelas correlações temporais.

PALAVRAS-CHAVE: Velocidade de vento; multifractais; MF-DFA.

References


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