ACCURACY OF THE UNIVARIATE F TESTS USED IN REPEATED MEASURES IN TIME WHEN THE SPHERICITY CONDITION OF THE COVARIANCE MATRIX IS NOT AVAILABLE

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ABSTRACT: In the analysis of repeated measures in time experiments using split-plot design, when the time is the sub-plot is a very common practice. Nevertheless it is not always correct, since this design requires the covariance Σ matrix with homogeneous structure. According to Huynh & Feldt (1970), the necessary and sufficient condition in order to have exact F test is that the Σ matrix satisfies the sphericity condition. On the other hand, Box (1954), Geisser & Greenhouse (1958) and Huynh & Feldt (1976) suggest approximated F tests, by correcting the degree of freedom, even the sphericity condition is not valid. The aim of this work is to evaluate the accuracy of these F tests approximated by simulation, for balanced or unbalanced data and different structures for covariance matrix Σ.

KEYWORDS: univariate analysis of variance, repeated measures in time, split-plot design, approximate F tests.

1 1 Introduction

Experiments with repeated measures in time involve, in general, 2 factors, as follows: treatments and times, and have as their main purpose to compare the treatments tendency along time, that is, if treatments profiles are horizontal and if they are parallel to each other.
Several techniques are available in literature for the analysis of such experiments. Those techniques must take into consideration the inter and intra times covariance matrix Σ structure. In practice, we find that for the same experimental unit, the observations are correlated, and such correlations are smaller for longer times, whereas the matrix structure is variable.
The univariate technique is a very common employed technique for the analysis of such experiments, suggested by Steel & Torrie(1980), which consists of considering a split-plot design, where "treatments" are plots and "times" are sub-plots. Nevertheless it is not always correct, since this design requires that the covariance matrix Σ to have homogeneous structure, what is not always verified.

According to Huynh & Feldt (1970), the necessary and sufficient condition in order to have exact F tests is that the matrix Σ satisfies the sphericity condition, that is:

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$$\sigma_{ij} = \frac{(\sigma_i^2 + \sigma_j^2)}{2} - \lambda \text{ , } \forall\ i \neq j \text{, and } \lambda \text{ a positive constant.}$$

Some authors such as Box (1954), Geisser & Greenhouse (1958) and Huynh & Feldt (1976), suggest some corrections on the degrees of freedom for the times factor and for the interaction between treatments and times, allowing this test to be employed in a approximated way, even though the sphericity condition is not satisfied.

The problem is that not much is known about the F tests accuracy when such corrections are performed, and whether the accuracy depends on the matrix Σ and the data to be balanced or not. Littell et al. (1998) suggest the correction proposed by Greenhouse & Geisser (1958).

The aim of this work is, using simulated data in computer, to avaluate the accuracy of the F tests for the variance analysis, for hypothesis of horizontal profile and profile's parallelism, in repeated measures in time experiments, when the correcting proposed by Greenhouse & Geisser (1959) and Huynh & Feldt (1976) are performed. Different structures of matrix Σ are considered (satisfying or not the sphericity condition), for balanced or unbalanced data.

2 Material and Methods

The data were simulated by a routine presented in Timm & Mieczkowski (1997). The data following a model of repeated measures in time experiment, that is, a maix of values $\boldsymbol{Y}=[y_{ijk}]$ with i=1, 2, ..., p (treatments), j=1, 2, ..., b (blocks) and k=1, 2, ..., t (times), in such way that the pb lines have t-varied normal distribution, are independents and that the covariance matrix between columns (Σ_{txt}) has one of the pre-established structures.

For the simulation, we have specified:

a) $p = 3$, $b = 4$ and $t = 5$;

b) Random effects for blocks, with mean zero and variance 25 ($bl \sim N(0,25)$);

c) Random effects for the treatments x blocks interaction, with mean zero and variance 36 (tr x $bl \sim N(0,36)$);

d) Null effect for treatments, times and for the interaction between treatments and times;

e) Two situation for the data balancing: balanced and unbalanced data (y_{111} e y_{142} lost);

f) Six structures for the matrix Σ, denoted by S_1, S_2, S_3, S_4, S_5 e S_6, as shown in Table 1. The structures S_1 and S_3 do satisfy the sphericity condition and the others do not.

Tabela 1 - Structures of covariance matrix Σ employed for the experiments simulation.

b.1) DEV

$$S_1=\begin{bmatrix}50&0&0&0&0\\0&50&0&0&0\\0&0&50&0&0\\0&0&0&50&0\\0&0&0&0&50\end{bmatrix}$$

b.2) DUV

$$S_2=\begin{bmatrix}10&0&0&0&0\\0&30&0&0&0\\0&0&50&0&0\\0&0&0&70&0\\0&0&0&0&90\end{bmatrix}$$

b.3) HS

$$S_3=\begin{bmatrix}50&25&25&25&25\\25&50&25&25&25\\25&25&50&25&25\\25&25&25&50&25\\25&25&25&25&50\end{bmatrix}$$

b.4) CTAP

$$S_4=\begin{bmatrix}50&20&0&0&0\\20&50&20&0&0\\0&20&50&20&0\\0&0&20&50&20\\0&0&0&20&50\end{bmatrix}$$

b.5) LDC

$$S_5=\begin{bmatrix}50&30&25&20&15\\30&50&30&25&20\\25&30&50&30&25\\20&25&30&50&30\\15&20&25&30&50\end{bmatrix}$$

b.6) AR

$$S_6=\begin{bmatrix}48&24&12&06&03\\24&48&24&12&06\\12&24&48&24&12\\06&12&24&48&24\\03&06&12&24&48\end{bmatrix}$$

Those structures were chosen for representing situations actually occurring, as follows: S_1: Diagonal with equal variances (DEV); S_2: Diagonal with unequal variances (DUV); S_3: Homogeneous structure (HS); S_4: Covariances only at times immediately anterior/posterior (CTAP) - TOEP(1) in the SAS classification ; S_5: Structure with linearly decrescent covariances (LDC) and S_6: First-order autoregressive structure (AR). One thousand assays were simulated for each case, and for each assay, the following tests are performed:

g) Matrix Σ sphericity test;

h) Univariate F tests in the following situations: with no correcting at all (usual analysis), with the correcting suggested by Greenhouse & Geisser (1959) and with the correcting suggested by Huynh & Feldt (1976).

Those tests were performed in the SAS, with the PROC GLM procedure and the REPEATED command, with the macro shown as follows:

```
%MACRO ANVPS(SDSE,CE);
DM 'LOG;CLEAR'; RUN; QUIT;
DM 'OUTPUT;CLEAR'; RUN; QUIT;
DATA SDSPS; SET &SDSE;
IF NOT(NE=&CE) THEN DELETE;
PROC GLM DATA=SDSPS;
CLASS I;
MODEL T1-T5=I/NOUNI;
REPEATED TP 5 POLYNOMIAL/PRINTE SUMMARY;
ODS OUTPUT Sphericity=T_esf;
```

```
ODS OUTPUT
GLM.Repeated.WithinSubject.ModelANOVA=T_GGHF;
PROC APPEND BASE=TESF DATA=T_ESF; RUN;
PROC APPEND BASE=TGGHF DATA=T_GGHF; RUN;
QUIT; QUIT;
%MEND ANVPS;
```

The chi-square value for sphericity test with respective Significance Minimum Level (SML), of each generated assay, was stored for study purposes. SML values of the F tests were stored for the following situations: with no correcting at all, with the correcting suggested by Greenhouse & Geisser (1959) and with the correcting suggested by Huynh & Feldt (1976).

From values obtained for the sphericity test, the averages were calculated, and from the SML related to such statistics, the percentage of significant tests at 5, 10 e 20% probability levels.

The SML values from F tests were distributed in frequency classes, in the range (0, 1), with 0,05 of amplitude. The F tests accuracy for the variance analysis was evaluated concerning such distribution, once according to MOOD *et al.*(1974), under the nullity hypothesis, in case the test requirements are satisfied, the SML values have uniform distribution in the range (0, 1).

The frequency distribution adherence to the uniform distribution was tested through the chi-square test.

3 Results and Discussion

Table 2 shows the average from the statistics obtained of the sphericity test of matrix Σ as well as the percentages of significant tests at 5, 10 e 20% probability level, concerning the six structures of matrix Σ (S_1, S_2, S_3, S_4, S_5 and S_6), for balanced or unbalanced data.

Tabela 2 - Average from the statistics of sphericity test of matrix Σ and the percentages of significant tests at 5, 10 e 20% probability level concerning the six structures of matrix Σ (S_1, S_2, S_3, S_4, S_5 and S_6), for balanced (Bal. =0) or unbalanced (Bal. =1) data.

Bal.	Statistics	Covariance matrix structure					
		S_1	S_2	S_3	S_4	S_5	S_6
	Average	0,3313	0,2041	0,3367	0,2038	0,2799	0,2339
	% Sig.(5%)	0,6	6,0	1,2	5,0	2,4	3,8
0	% Sig.(10%)	4,6	22,0	4,6	24,2	9,6	15,8
	% Sig.(20%)	21,4	55,2	24,8	54,2	37,2	47,6
	% N.Sig.(20%)	78,6	44,8	75,2	45,8	62,8	52,4
	Average	0,2339	0,1437	0,2316	0,1525	0,1989	0,1716
	% Sig.(5%)	8,0	21,4	8,4	20,2	13,6	17,6
1	% Sig.(10%)	20,2	45,6	23,2	44,0	30,6	38,8
	% Sig.(20%)	49,4	75,0	47,4	71,8	58,4	66,0
	% N.Sig.(20%)	50,6	25,0	52,6	28,2	41,6	34,0

One observes through Table 2 that, for balanced data and homogeneous structures (S_1 and S_3), the obtained averages for the sphericity test chi-squared are near to one another (approximately 0,33) and greater than the other structures (near to 0,20 for S_2, S_4 and S_6, and near to 0,28 for S_5). The superiority for S_1 and S_3 was expected once those structures do satisfy the sphericity condition, and the other studied structures do not. For unbalanced data, those averages are always smaller than the averages obtained for balanced data, for the corresponding cases, and for S_1 and S_3, are near to each other (approximately 0,23) and greater than the other structures (near to 0,15 for S_2, S_4 e S_6, and 0,20 for S_5).

Through percentages of significant tests at 20% probability level, one also observes that, for balanced data and for the non-spherical structures: S_2, S_4, S_5 and S_6, approximately 50% from the sphericity tests are significant at 20% probability level, with an outstanding variation for S_5 (37,2%), and for the spherical structures (S_1 e S_3), approximately 20% from the sphericity tests are significant at 20% probability level. For unbalanced data, such percentages increase quite much; regardless the structures of the matrix Σ, following the same verified superiority relation for the balanced data.

Such results allow us to conclude that the sphericity tests are sensible to the absence of balancing and, regardless data are balanced or not, the sphericity tests show variations depending on the covariance matrix structure.

Table 3 shows the average of the chi-squared statistics values obtained from the frequency distribution adherence test of the significance minimum levels (SML) related to the F statistical values for times (*Tp)* and for the interaction between treatments and times (*Tr* x *Tp)*, concerning the six structures of the matrix Σ (S_1, S_2, S_3, S_4, S_5 and S_6), and balanced or unbalanced data.

Tabela 3 - Averages of the chi-squared statistics values obtained from the frequency distribution adherence test of the significance minimum levels related to the F statistical values for times *(Tp)* and for the interaction between treatments and times (*Tr* x *Tp)*, concerning the six structures of the matrix Σ (S_1, S_2, S_3, S_4, S_5 and S_6), and balanced (Bal.=0) or unbalanced (Bal.=1) data.

Variation sources	Bal.	Statistic F	Variance and Covariance matrix structure					
			S_1	S_2	S_3	S_4	S_5	S_6
Tp	0	No Cor.	14,32	22,48	12,96	**32,56**	14,24	**32,64**
		Cor. HF	**43,60**	**59,28**	**40,78**	**51,76**	**51,12**	**50,16**
		Cor. GG	15,56	19,76	11,44	27,84	14,00	25,44
	1	No Cor.	26,08	30,00	12,32	**35,12**	17,36	26,16
		Cor. HF	**68,48**	**67,20**	**58,24**	**56,08**	**74,16**	**61,68**
		Cor. GG	25,76	29,12	11,52	27,44	25,12	25,12
Tr x *Tp*	0	No Cor.	14,56	19,20	12,88	13,36	12,00	14,96
		Cor. HF	**49,68**	**38,72**	**43,92**	**33,60**	**49,52**	**43,68**
		Cor. GG	17,04	28,96	14,80	13,68	13,04	11,84
	1	No Cor.	11,84	22,80	17,04	14,16	16,56	12,48
		Cor. HF	**63,28**	**60,80**	**59,28**	**61,28**	**54,24**	**67,84**
		Cor. GG	14,06	20,00	16,24	14,48	21,28	13,28

No cor.: With no correcting; **Cor. HF:** Huynh & Feldt (1976) correcting; **Cor. GG**: Greenhouse & Geisser (1958) correcting.

According to the concerned number of frequency classes, which were 20, the adherence hypothesis tested through the chi-squared test is rejected at 5% probability level, if the statistical value is greater than 30,14. Thus, the following is observed from this Table:

i) For F tests with no correcting at all, the adherence hypothesis is not rejected in general, at 5% of probability level, except for times tests (*Tp*) with balanced data, for the structures S_4 and S_6, and with unbalanced data for the structure S_4;

j) For F tests with the Huynh & Feldt (1976) correcting, the adherence hypothesis is rejected at 5% of probability level, for both times (*Tp*) and the interaction (*Tr* x *Tp*) for all studied cases, regardless data being balanced or not as well as the structure of the matrix Σ;

k) For F tests with the Greenhouse & Geisser (1959) correcting, the adherence hypothesis is not rejected at 5% of probability level for both times (*Tp*) and the interaction (*Tr* x *Tp*) for all studied cases, regardless data being balanced or not as well as the structure of the matrix Σ;

Such results allow us to conclude that for both times (*Tp*) and for the interaction (*Tr* x *Tp*), the F tests from the variance analysis are accurate with the Greenhouse & Geisser (1959) correcting and with the Huynh & Feldt (1976) correcting, they are not, regardless data being balanced or not as well as the structure of the matrix Σ. The results agree with Littell et al.(1998), which suggest the Greenhouse & Geisser (1959) correcting as the best choice.

When no correcting at all is made, the F tests for the interaction *Tr* x *Tp* show to be accurate, and for times (*Tp*), the accuracy depends on the structure of the matrix Σ.

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RESUMO: A análise de experimentos com medidas repetidas no tempo usando o esquema em parcelas subdivididas, tendo o tempo como sub-parcelas, é prática comum, mas nem sempre é correta pois, como se sabe, este esquema pressupõe que a matriz de covariâncias Σ *tenha uma estrutura homogênea, o que nem sempre é verificado. Segundo Huynh & Feldt (1970) uma condição necessária e suficiente para que os testes F dessa análise sejam exatos é que a matriz* Σ *satisfaça a condição de esfericidade. Alguns autores como Box (1954), Geisser & Greenhouse (1958) e Huynh & Feldt (1976), sugerem correções nos graus de liberdade, para que esses testes possam ser usados de forma aproximada, mesmo que a condição de esfericidade não seja satisfeita. Neste trabalho avalia-se, através de dados simulados em computador, a precisão dos testes F, quando essas correções são realizadas, e se a precisão depende da estrutura da matriz de covariâncias* Σ *e dos dados serem balanceados ou não. Observa-se melhor precisão para a correção de Geisser & Greenhouse (1958).*

PALAVRAS-CHAVE: análise de variância univariada, medidas repetidas no tempo, esquema em parcelas subdivididas, testes F aproximados

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